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Development of Methodology for Design and Analysis of Physically Cooperating Robot and Applications to Other Robotic Systems

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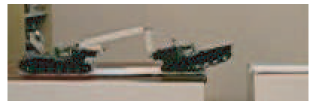
***Abstract*— There is a need to develop a methodology to model physically cooperating mobile robots so as to systematically design and analyze such systems. Our approach is to treat the linked mobile robots as a multiple degree-of-freedom object, comprising an articulated open kinematic chain, which is being manipulated by *pseudo robots* (*p-robots*) at the ground interaction points. Dynamics of the open chain are computed independently of the constraints, thus allowing the same set of equations to be used as the constraint conditions change, and simplifying the addition of multiple robots to the chain. The decoupling achieved through constraining the *p-robots* facilitates the analysis of kinematic as well as force constraints, not possible with direct analysis. We introduce the idea of a 'tipping cone', similar to a standard friction cone, to test whether forces on the robots cause undesired tipping. We have carried out static as well as dynamic analysis for a 2-robot cooperation case. Also, we have demonstrated that introduction of redundant actuation, by an additional third robot, can help in improving the friction requirements. We also present our preliminary ideas for employing this newly designed framework to analyze other interesting multi-body robotic systems.**

I. Introduction

Mobile robots are useful for applications such as search and rescue, urban infiltration etc, where the goal is to explore unknown, potentially hazardous terrains. Large teams of small, cheap robots have advantages over small numbers of large, expensive robots, such as covering more ground in less time, access to tight spaces, redundancy, and expendability. One of the major challenges in employing small mobile robots is their restricted mobility on rough terrain. Physical cooperation among robots is proposed as one method to overcome the mobility restrictions.

A. Physical Cooperation for Mobility Improvement

A number of researchers have proposed and developed teams of robots in which team members physically cooperate to improve mobility. Hirose *et.al.* [10], [13] have developed a chain of mobile robots inspired by snake motion to inspect hazardous areas of a nuclear plant. Team of small robots (each around 6 cm long), called Millibots [5] is another example of physically cooperating robots. Researchers at EPFL, Switzerland have demonstrated impressive cooperative behaviors with robots (each around 15 cm long) possessing multiple sensors and actuators [15]. Another example is the group of robots developed by Asama *et. al.* [1] which cooperate via a forklift mechanism to climb steps. In our earlier work [7], [6], [8], [9] we have proposed a team in which cooperation is achieved through un-actuated linkages. Figure 1 shows such a system in action. Such systems with no additional link actuation as opposed to other approaches, such as fork lift mechanism [1], lead to lower cooperation costs, ability to retrofit cooperative behaviors on existing mobile robots, and scalability.



(a) Stage 1 of gap crossing

(b) Stage 2 of gap crossing

(c) Stage 3 of gap crossing

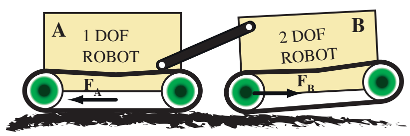
(d) Stage 4 of gap crossing

Fig. 1. Snap shots from the demonstration of gap crossing with physical cooperation [6]. The robots are connected by a link hinged at each robot and forces are applied at the robot wheels to lift and move the robots. There is no additional actuator at the link and all forces come from the wheel torques.

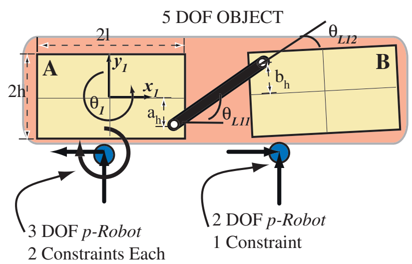
Recognizing that small robots need to overcome discrete obstacles we have developed specific maneuvers to negotiate each obstacle, for example, the behaviors developed to overcome a discrete obstacle (a long gap) with physical cooperation between two robots [6], [8], [9]. This is a systematic approach to mobility improvement as opposed to having the robot pair scramble over obstacles. We want to generalize the ideas of cooperative mobility improvement to achieve a larger class of maneuvers involving multiple robots to traverse rougher terrain and more extreme obstacles. However, to go through detailed analysis for each and every behavior is cumbersome and as the number of robots increases it is impractical due to multi-fold increase in the complexity of the system: it is laborious to develop a dynamic model of connected robots with direct analysis and then the model is too complex to be useful. We want to develop a methodology to efficiently develop dynamic models of physically connected robots with a structure that is suitable for design and analysis. For example, we want to develop a method to efficiently derive a complete dynamic model of two connected robots to design motions under given friction conditions, while enabling the easy addition of more robots in the analysis.

B. General Idea for a Representation

Although there are a number of teams of robots physically cooperating to improve *mobility*, currently, there does not exist a methodology for systematic design and analysis of such systems. Juxtaposing to the field of robotic cooperation for *mobility* is the field of robotic cooperation for *manipulation*



(a) Two robots connected by a simple link



(b) An equivalent system with an articulated object and manipulators

Fig. 2. Two-robot cooperation. The 2-robot chain is considered as a 5-DOF articulated object which is acted upon by two manipulators or *pseudo robots* (*p-robots*). Additional generalized coordinates are introduced to bring out the interaction/constraint forces.

and there has been a considerable effort toward the development of representations for such systems. We plan to extend and adopt these approaches to develop a framework for robots cooperating to improve mobility.

The general idea is to treat the system of cooperating mobile robots as a equivalent manipulation system. We treat the chain of robots, whose motions we want to control, as one object and analyze the situation as the wheels or treads manipulate this *object* via interaction with ground. For example in the case of Figure 2, the chain of two robots is treated as a manipulated object and the wheels at the bottom act as the manipulators.

C. Representations for Cooperative Manipulation

Development of framework to represent a system of robots that cooperate to manipulate a common object is a well-studied field of research [19], [4], [20], [3], [17]. We consider three distinct approaches to representation: Bicchi's [4], [2] method to characterize mobility and differential kinematics of general cooperating systems; the model proposed by Williams and Khatib[20], [14] to characterize internal forces and moments during a multi-grasp manipulation; and the approach proposed by Srinivasa, Mason and Erdmann [17], [18] to relate the kinematic (velocity) constraints as well as the force constraints to the dynamics of the object. It is obvious that these formal approaches developed for the cooperative *manipulation* problems can not directly be translated to the case of robot cooperation for *mobility*. However, these approaches can form building blocks for the development of a modified representation of systems that cooperate to improve mobility. Moreover, these approaches address three different issues of cooperative manipulation, namely, the kinematic analysis, static force analysis and dynamical analysis; and there is scope to develop a unified approach based on the available results.

First, in Section II we describe our approach in detail and present the methodology with an example of the 2-robot cooperation case. In Section III we implement the newly developed representation to the 2-robot example to carry out static as well as dynamic analysis. We also demonstrate, with the 3-robot example, the introduction of redundant actuation to reduce friction requirements. In Section IV we present our preliminary ideas for employing this representation to analyze other interesting robotic systems. Finally, we conclude and present the direction for future work in Section V.

II. Representation for Cooperative Mobility

To adopt above mentioned approaches to analyze cooperative mobility, we need to first re-structure the problem of analyzing cooperative mobility systems. Our idea for this is to treat the chain of robots as an *object* and the wheels/treads as the manipulating *pseudo robots* (we refer to these as *p-robots*). Consider the case shown in Figure 2 where two robots are connected by a hinged link and are cooperating to lift one end of a robot. With our approach we consider two connected robots as one articulated object manipulated by two point *p-robots*. We present the development of our approach with the help of this 2-robot example.

A. Constraint Forces and Generalized Coordinates

In many of the above listed dynamic manipulation approaches internal or constraint forces do not appear explicitly in the equations of motion (EOMs) and hence are not determined. In case of cooperative mobility systems, many times it is necessary to determine some of the constraint forces, *eg*. We can do this by 'pseudo' motions to the *pseudo robots*, for each interaction force to be determined and then adding a velocity constraint to the system. This is based on the 'Lagrange multiplier' technique used in case of energy based methods for dynamic equation generation [11], [12]. Similar idea is used by Yamane and Nakamura [21] to determine constraint forces in case of dynamic motion generation of human figures. Here we present methods to determine two types of constraint forces that are critical for cooperative mobility problems.

1) Normal Forces and Friction Cone: For the various physical cooperative maneuvers mentioned in [6], [13], [5] the desired motions are generated through interactions with the ground. Therefore, in order to design and analyze such cooperative motions, we need to determine ground interaction forces. As mentioned in [6] it is critical to satisfy the friction conditions so that robots do not slip while cooperating.

Consider the 2-robot case. Cooperation is achieved via actuation only at the wheels, and the resultant forces on the robots must lie within the friction cone defined by the coefficient of friction μ at the point of contact between the tracks and the ground:

$$F_{aX} < \mu F_{aY} \quad \text{and} \quad F_{bX} < \mu F_{bY}$$

where F_{aX} and F_{aY} are the horizontal and the vertical forces on robot A, and F_{bX} and F_{bY} are the horizontal and the vertical forces on the robot B as shown in Figure 2(b). Figure 3(a) shows the friction cone along $F_X - F_Y$ axes. To satisfy the friction requirement forces must lie within this cone. Equivalently this condition can be represented as[1]:

$$\mathbf{f_a} \in \mathcal{F}_A \quad \text{and} \quad \mathbf{f_b} \in \mathcal{F}_B$$

where $\mathbf{f_a}$ and $\mathbf{f_b}$ are the resultant forces at the contact point on robots A and B, and $\mathcal{F}_A$ and $\mathcal{F}_B$ are the friction cones.

In order to test this condition using our representation we need to determine the normal forces at the contact ground interaction points. To achieve this we define two 'auxiliary' robot generalized coordinated: y_{r1} and y_{r2} representing the vertical motion of the two robots. The corresponding contact generalized coordinates are y_{c1} and y_{c2}.

2) Moment and Tipping Cone: In the 2-robot case the robots cooperate to lift up one end of a robot B as shown in Figure 2. We do not want robot A to lift up instead. We want to develop a test to check whether the moment acting on the robots causes unwanted tipping. In the 2-robot case one can check for tipping by determining the moment and the vertical force on robot A.

A vertical force and moment pair at one point can be considered as equivalent to vertical force and moment pair at another point according to following relationship:

$$\{f_{yO}, M_O\} \equiv \{f_{yD}, M_D\}$$

where

$$f_{yO} = f_{yD} \quad \text{and} \quad M_D = M_O - (\overline{OD})_x f_y$$

where $(\overline{OD})_x$ is the horizontal distance between points O and D on robot A. Any force-moment pair can be shifted horizontally where the equivalent pair has zero moment.

$$\{f_{yO}, M_O\} \equiv \{f_{yt}, 0\}$$

The condition for robot A staying on the ground is that the line of action of the vertical force f_{yt} with zero moment has to lie within the robot's body length, $(2l)$.

$$\begin{aligned} -l < (\overline{OT})_x &= \frac{M_O}{f_{yO}} < l \\ &\text{OR} \\ -1 < tp &\equiv \frac{M_O}{f_{yO} l} < 1 \end{aligned} \tag{1}$$

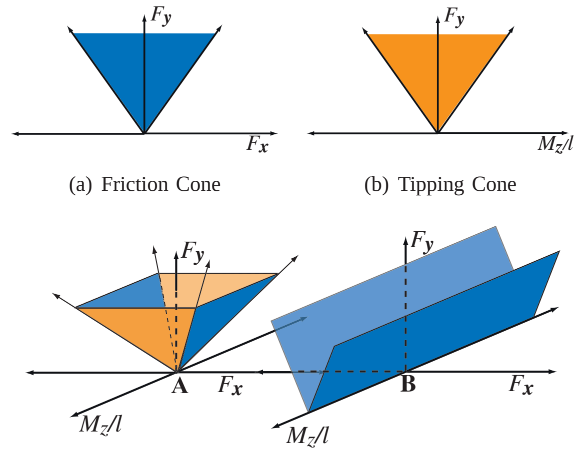
This tipping condition tp is analogous to the friction condition, and we can define a tipping cone similar to the friction cone but in the $F_Y - M/l$ plane, as shown in Figure 3(b). To satisfy the tipping condition, forces must lie within this cone. Equivalently this condition can be represented as

$$\frac{M_{AZ}}{l} \in \mathcal{M}_A$$

where $\mathcal{M}_A$ is the tipping cone. To determine the moment on robot A we introduce an auxiliary robot coordinate θ_{r1} and corresponding contact auxiliary coordinate θ_{c1}.

Note that the shape of the 'Tipping Cone' depends on the location of the *p-robot* along the length of body A, which is arbitrary. To get the cone symmetric about the f_y axis we choose the location of the *p-robot* at the body center.

[1]The conventions and notations used here are similar to those in [17], [18].



(a) Friction Cone (b) Tipping Cone

(c) Combined Cones

Fig. 3. Friction and tipping cones: The resultant force must lie within the friction cone to avoid slipping, and the vertical force and resultant moment must lie within the tipping cone to avoid tipping. The combined cone is formed by the intersection of the friction and tipping cones. Only the rear robot A has a 3-D cone since there is no tipping condition for robot B.

3) Combined Cone: We can combine the friction and the tipping cones in the space $F_X - F_Y - M/l$ as shown in Figure 3(c) where the combined cone is the intersection of the friction and the tipping cones. To satisfy the friction and the tipping condition the forces and moments must lie inside of the combined cone. Note that only the rear robot A has a 3-D combined cone and combined cone at robot B is 2-D since there is no tipping condition for this robot.

Note that this approach of adding 'pseudo' generalized coordinates to determine constraint forces can be extended to determine other internal/constraint forces (*e.g.* link tension).

B. Methodology with Two Robot Example

For the two robot case as shown in Figure 2 the *object* is an open chain free flowing in 2D space, composed of two rigid bodies connected by a massless link and hence has five degrees of freedom. Keeping above discussion in mind lets define variables for the two robot case as follows: object generalized coordinates: $\mathbf{q_o} = [x_o, y_o, \theta_{o1}, \theta_{l11}, \theta_{l12}]^T$, as represented in the Figure 2(b), robot generalized coordinates: $\mathbf{q_r} = [x_{r1}, y_{r1}, \theta_{r1}, x_{r2}, y_{r2}]^T$, which represent x-y positions of two *p-robots* and rotation of *p-robot* A. This is a two degree of freedom system and those are defined as: $\mathbf{q_{DOF}} = [x_{RR}, \theta_{FR}]^T$. We can also define the contact generalized coordinates as: $\mathbf{q_c} = [x_{c1}, y_{c1}, \theta_{c1}, x_{c2}, y_{c2}]^T$. In case of more robots, we will have more object as well robot degrees of freedom and they can be laid out in similar fashion.

1) Kinematic Relations: The relation between the robot generalized velocities and the contact generalized velocities, and that between the object generalized velocities and the contact generalized velocities is given by:

$$\dot{\mathbf{q}}_\mathbf{c} = J\dot{\mathbf{q}}_\mathbf{r}; \qquad \dot{\mathbf{q}}_\mathbf{c} = G\dot{\mathbf{q}}_\mathbf{o} \tag{2}$$

where J is the robot Jacobian matrix which relates the robot joint velocities to the 'end effector' velocities and G is the object 'grasp matrix' which maps the forces on the object at the contact point to wrenches about the object's center of gravity. In the two robot example since the two *p-robots* are simple point elements the robot and contact velocities are the same, the Jacobian is an identity matrix with dimensions 5×5. G is equivalent to the Jacobian of the 2-robot open chain which can easily be determined and has dimensions 5×5. Note that J and G do not assume any constraint with respect to ground. We can establish a fixed contact kinematic constraint[2] as:

$$J\dot{\mathbf{q}}_{\mathbf{r}} = G\dot{\mathbf{q}}_{\mathbf{o}} \tag{3}$$

As mentioned in the earlier section, we add auxiliary robot coordinates to the system to determine the constraint forces, and we add constraints that set these auxiliary robot velocities to zero. Since we have added auxiliary coordinates to the *p-robots* the auxiliary constraints have the form:

$$0 = J_a\dot{\mathbf{q}}_{\mathbf{r}}. \tag{4}$$

where J_a defines constraint relationship on *p-robots*. In the 2-robot case there are three *p-robot* auxiliary constraints: $\dot{y}_{r1} = 0$, $\dot{\theta}_{r1} = 0$ and $\dot{y}_{r2} = 0$ and thus J_a is a 3×5 matrix.

Note that Srinivasa *et.al.* [17] deal with the interaction of the object with the environment by adding environment constraints to the object velocity equations. We choose to add auxiliary generalized coordinates and constraints to the *p-robot* coordinates and we will see in the following analysis that setting constraints on the *p-robots* instead of on the object leads to useful decoupling of the problem.

To incorporate the robot auxiliary constraints we augment the matrices J and G, and re-write equation 3 as follows:

$$J_c = \begin{bmatrix} J_a \\ J \end{bmatrix}; \quad G_c = \begin{bmatrix} 0 \\ G \end{bmatrix} \quad \Rightarrow J_c\dot{\mathbf{q}}_{\mathbf{r}} = G_c\dot{\mathbf{q}}_{\mathbf{o}} \tag{5}$$

We can determine the degrees of freedom of the system as: $n_{\mathbf{DOF}} = [n_{\mathbf{q_o}} + n_{\mathbf{q_r}} - \text{no. of constraints}]$. The 2-robot case, with 5 object generalized coordinates, 5 robot generalized coordinates and total of 8 constraints (including auxiliary constraints), leads to a total of 2 DOF.

2) Dynamic Relations: The next step is to generate the unconstrained dynamic equations of motions of the object. For the 2-robot case, the object is an articulated chain with 2 rigid bodies connected by a massless link, that has 5 DOFs, given by: $\mathbf{q_o} = [x_o, y_o, \theta_{o1}, \theta_{l11}, \theta_{l12}]^T$. The equations of motion for such an open chain can be generated by using any standard dynamics method such as Lagrange Method [11]. Such equations are widely derived [16] with a standard form:

$$M(\mathbf{q_o})\ddot{\mathbf{q}}_{\mathbf{o}} + C(\mathbf{q_o}, \dot{\mathbf{q}}_{\mathbf{o}})\dot{\mathbf{q}}_{\mathbf{o}} = \tau - G^T(\mathbf{q_o})\mathbf{f_c} + \mathbf{g}(\mathbf{q_o}) \tag{6}$$

where $\mathbf{M}(\mathbf{q_o})$ is the inertia matrix, $\mathbf{C}(\mathbf{q_o}, \dot{\mathbf{q}}_{\mathbf{o}})$ is a matrix that represents the Coriolis terms, $\mathbf{g}(\mathbf{q_o})$ is a vector that represents the gravity terms, G^T represents the transpose of the grasp matrix which is defined above, τ represents the joint torques vector which is zero in our case, and $\mathbf{f_c}$ represents the external force on the object *i.e.* those applied by the manipulating robots on the object at the contact point. In the 2-robots case these are given as: $\mathbf{f_c} = [F_{aX}, F_{aY}, M_{aZ}, F_{bX}, F_{bY}]^T$. Note that the dynamics equations need only to be generated for the open chain rather than for a closed chain of multi-body system as would be necessary in the direct derivation approach.

3) Combining the Constraints: The kinematic constraints, including the auxiliary ones (represented by equation 5), and the dynamic constraints can be incorporated into the system in the acceleration domain. We first take time derivative of equation 5 and then combine it with equation 6 to give:

$$\begin{aligned} J_c\ddot{\mathbf{q}}_{\mathbf{r}} - \dot{G}_c\dot{\mathbf{q}}_{\mathbf{o}} + \dot{J}_c\dot{\mathbf{q}}_{\mathbf{r}} + G_cM^{-1}C\dot{\mathbf{q}}_{\mathbf{o}} - \\ G_cM^{-1}\mathbf{n_o} = -G_cM^{-1}G^T\mathbf{f_c} \end{aligned} \tag{7}$$

Based on the forms of matrices J_c and G_c given by equation 5, equation 7 can be decoupled into two as follows:

$$J_a\ddot{\mathbf{q}}_{\mathbf{r}} - \dot{J}_a\dot{\mathbf{q}}_{\mathbf{r}} = 0 \tag{8}$$

$$\begin{aligned} J\ddot{\mathbf{q}}_{\mathbf{r}} - \dot{G}\dot{\mathbf{q}}_{\mathbf{o}} + \dot{J}\dot{\mathbf{q}}_{\mathbf{r}} + GM^{-1}C\dot{\mathbf{q}}_{\mathbf{o}} - \\ GM^{-1}\mathbf{n_o} = -GM^{-1}G^T\mathbf{f_c} \end{aligned} \tag{9}$$

Such decoupling is advantageous since equation 8 is a purely kinematical relation which represents the robot auxiliary constraints in the acceleration domain and equation 9 represents the constrained dynamics of the robot-object system. In the 2-robots case equation 8 gives 3 relations and equation 9 gives 8 relations.

Defining $\widetilde{G} = (G^T)^{-1}MG^{-1}$, equation 9 becomes

$$\widetilde{G}J\ddot{\mathbf{q}}_{\mathbf{r}} - \widetilde{G}\dot{G}\dot{\mathbf{q}}_{\mathbf{o}} + \widetilde{G}\dot{J}\dot{\mathbf{q}}_{\mathbf{r}} + (G^T)^{-1}C\dot{\mathbf{q}}_{\mathbf{o}} - (G^T)^{-1}\mathbf{n_o} = \mathbf{f_c} \tag{10}$$

where $\mathbf{f_c}$ is a force vector that includes the applied forces as well as the constraint forces. We have discussed in the earlier section that the forces have to follow the friction cone as well as the tipping cone constraints. Now we can test force constraints conditions as:

$$\widetilde{G}J\ddot{\mathbf{q}}_{\mathbf{r}} + V(\dot{\mathbf{q}}_{\mathbf{o}}, \dot{\mathbf{q}}_{\mathbf{r}}, \mathbf{n_o}) \in \mathcal{F} \tag{11}$$

where

$$V(\dot{\mathbf{q}}_{\mathbf{o}}, \dot{\mathbf{q}}_{\mathbf{r}}, \mathbf{n_o}) = -\widetilde{G}\dot{G}\dot{\mathbf{q}}_{\mathbf{o}} + \widetilde{G}\dot{J}\dot{\mathbf{q}}_{\mathbf{r}} + (G^T)^{-1}C\dot{\mathbf{q}}_{\mathbf{o}} - (G^T)^{-1}\mathbf{n_o}$$

and $\mathcal{F}$ is the combined force constraint cone. For the 2-robots,

$$\mathcal{F} = (\mathcal{F}_A \times \mathcal{M}_A) \times \mathcal{F}_B$$

Since we were able to decouple equation 7 we can test the force constraints in the force domain itself rather than in the contact acceleration domain as done in [17], [18]. This is possible because we set auxiliary constraints on the manipulating robots rather than on the object. Of course, analysis in the force domain is possible only when G^{-1} exists. G is the grasp matrix of the articulated object with the same conditions for existence of its inverse as the Jacobian of an open chain robot [16], which are discussed extensively in the literature. Also, equation 7 is similar to equation 17 in [21], however, our method of checking the force constraint condition is very different.

This approach is advantageous in a number of ways. Since we generate dynamic equations only for an open chain this process is easier and we can use standard method derived in

[2] Any other type of contact condition can be handled by introducing a contact constraint matrix H which leads to kinematic constraint $H(J\dot{\mathbf{q}}_{\mathbf{r}} - G\dot{\mathbf{q}}_{\mathbf{o}}) = 0$, as done by Bicchi [4]

the robotics literature for this purpose. It is easier to implement constraints on the contact coordinates rather than implementing multi-body loop constraints as would be necessary with a direct analysis. Also, addition of more robots to the analysis is much easier with our approach.

III. Analysis with 2-Robot and 3-Robot Chains

In this section we present the implementation of the methodology developed in the earlier section to analyze 2-robot and 3-robot chains. In our previous work [6], we carried out direct analysis of 2-robot cooperation, and then designed and developed a hardware platform to test our results. Below we carry out analysis of the same system with our newly developed method.

A. Two Robot Example

A detailed analysis of a 2-robot chain with conventional energy based methods is presented in [6]. Here we demonstrate that our representation can be employed to obtain the same results when analyzing the cooperative lifting of one end of pair of robots.

1) Static Analysis: We determine the forces and friction required to quasi-statically lift the front end of robot B of the pair of robots shown in Figure 2(a) using using our newly developed representation by setting all the velocities and accelerations to zero. Equations 8 and 9 reduce to $(G^T)^{-1}\mathbf{n_o} = \mathbf{f_c}$, and solving this equation yields

$$-F_{aX} = F_{bX} \geq \frac{mgl}{(h + b_h)} \tag{12}$$

where m is the mass of each robot, l is the robot half length, h is the robot half height, b_h is the link connection point on robot B as shown in Figure 2(a). So, for lifting to occur equal and opposite traction forces on robots A and B must be higher than the limit derived in the above equation.

The ground friction requirements for robots A and B are

$$\mu_B \geq \frac{l}{(h + b_h) + l\tan(\theta_{l11})}; \quad \mu_A \geq \frac{l}{(h + b_h) - l\tan(\theta_{l11})} \tag{13}$$

where θ_{l11} is the angle of the link relative to robot A as shown in Figure 2(a). These conditions match with those presented in the previous work which were derived by simple force balance [6]. Since the hardware system was developed based on the direct analysis results and since the new methodology gives the same results as the direct analysis, the hardware results are proof of concept for new methodology as well.

With the same configuration and parameter values as used in [6] we get: $-F_{aX} = FbX > 314.44$ N, $\mu_A > 1.344$, $\mu_B > 0.571$. The friction requirement for robot A forms the critical limit, and for the configuration considered it is much higher than practically achievable. Thus this analysis tells you that we need to either re-design the configuration or, as proposed in [6], use robot dynamics to relax the requirement.

We can also determine the tipping condition for robot A by solving the equation $(G^T)^{-1}\mathbf{n_o} = \mathbf{f_c}$. For the example case, the tipping condition is $tp_A = -0.210$ which is well within the allowable limit of ± 1.

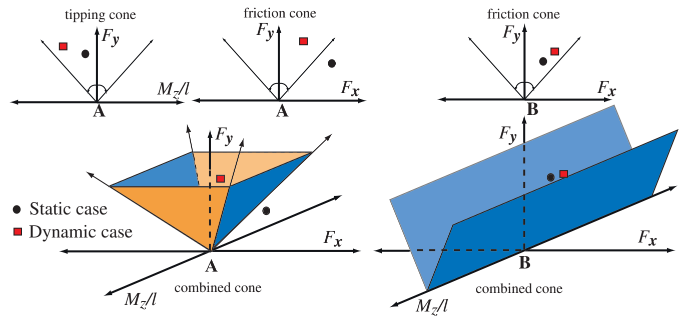


Fig. 4. Friction and Tipping Cone for the 2-robot case (not to the scale): The combined cones at robots A and B are formed by the intersection of the friction and the tipping cones. The friction and tipping conditions change as dynamics are introduced. For robot A under the static conditions the friction requirement is very high (outside of the friction cone) but it is brought inside of the cone with the introduction of dynamics. The tipping condition on robot A and the friction requirement on robot B stay inside of the respective cones for the static as well as the dynamic lifting case.

Figure 4 gives the combined friction and tipping cone for the robot A and the friction cone for the robot B. For the example problem, for the static case the friction requirement on robot A is outside of the friction cone, but the tipping condition on robot A and the friction requirement on robot B are both well inside of the respective cones.

2) Introducing Dynamics in 2-robot Lifting Analysis: In [6], it is proposed that friction requirements for the front robot lifting can be reduced by dynamic lifting *i.e.* accelerating the pair of robots as the front robot is lifted, instead of quasi-static lifting. This hypothesis comes from the fact that a single vehicle can not pop-up its own front wheels statically but can do so if accelerating. For the 2-robot case, dynamics are introduced if we apply unequal opposing traction forces on the robots. We want to determine, using the newly developed representation, what is the best acceleration from a dead stop to achieve lifting even on low friction surfaces. Setting all velocities to zero reduces equations 8 and 9 to:

$$J_a \ddot{\mathbf{q}}_\mathbf{r} = 0 \tag{14}$$

$$\widetilde{G} J \ddot{\mathbf{q}}_\mathbf{r} + (G^T)^{-1}\mathbf{n_o} = \mathbf{f_c} \tag{15}$$

Equation 14 gives a constraint equation on robot accelerations to set $\ddot{y}_{r1}, = \ddot{\theta}_{r1}, = \ddot{y}_{r2}, = 0$. We introduce dynamics by accelerating robots along the x-direction by setting non-zero values for $\ddot{x}_{r1}$ and $\ddot{x}_{r2}$. We set $(\text{accln})_x = \ddot{x}_{r2} = \ddot{x}_{r1}$ so that the two robots accelerate together and the front robot barely lifts. Under these conditions we can vary the acceleration on the pair of robots $(\text{accln})_x$ to reduce the friction requirement.

In the 2-robot case horizontal acceleration is introduced by applying non-equal traction forces on the two robots. Figure 5 shows the variation of the friction requirements on the two robot as the ratio of traction forces on the two robots, defined as $f_r = F_{aX}/F_{bX}$, varies. Note that $f_r = 1$ represents the static case. The figure shows that as f_r increases, μ_A goes down and μ_B goes up. The two curves intersect to give an optimal friction requirement as shown in the figure.

Figure 4 shows how both the friction and tipping conditions change as dynamics are introduced in the system. For robot A

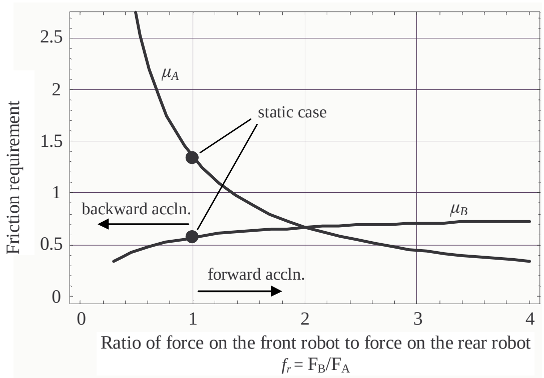


Fig. 5. Exploiting dynamics to relax the friction requirement: The friction requirements on the two robots change as the ratio of traction forces f_r on the two robots varies. $f_r = 1$ is the static lifting case. As f_r increases *i. e.* as the robots accelerate forward while lifting, the overall friction requirement goes down.

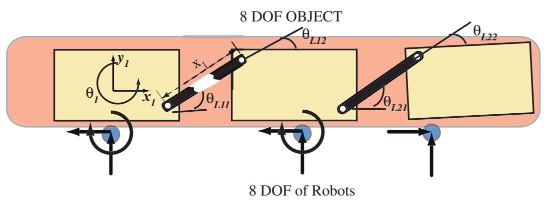


Fig. 6. The three robot manipulation system cooperates to lift one end of robot C. The 3-robot chain is considered as a 7-DOF articulated object which is acted upon by three manipulators or *p-robots*. An additional *virtual linkage* DOF is added through the link connecting robots A and B.

under static conditions, the friction requirement is very high (outside of the friction cone) but it is brought inside with the introduction of dynamics. The tipping condition on robot A and the friction requirement on robot B stay inside of the respective cones for both the static and dynamic lifting cases.

B. Redundant Actuation: 3-robot Cooperation

Until now we have considered cooperation between only two robots to achieve the desired maneuvers, where high ground friction is necessary to lift up one end of the pair of robots and that a dynamic maneuver can reduce the friction requirement. Another solution is to add one more robot in the chain and use the redundant actuation to distribute the traction forces among the three robots. In this section we analyze 3-robot cooperation to lift the front end of the front robot and demonstrate that our representation handles the addition of a robot and is useful to determine the optimal distribution of traction forces to reduce the ground friction requirement.

As shown in Figure 6, we define generalized coordinates for the 3-robot case for the object, $\mathbf{q_o} = [x_o, y_o, \theta_{o1}, \theta_{l11}, \theta_{l12}, \theta_{l21}, \theta_{l22}]^T$, the robot, $\mathbf{q_r} = [x_{r1}, y_{r1}, \theta_{r1}, x_{r2}, y_{r2}, \theta_{r2}, x_{r3}, y_{r3}]^T$, and for the contact points $\mathbf{q_c} = [x_{c1}, y_{c1}, \theta_{c1}, x_{c2}, y_{c2}, \theta_{c2}, x_{c3}, y_{c3}]^T$. This forms a redundant system for the move and lift maneuver.

Figure 6 shows the 3-robot system. Note that there are 7 object generalized coordinates and 8 *p-robot* coordinates which present a redundant system *viz.* 8 contact forces $\mathbf{f_c} = [F_{aX}, F_{aY}, M_{aZ}, F_{bX}, F_{bY}, M_{bZ}, F_{cX}, F_{cY}]^T$ that affect 7 object generalized coordinates. Our idea is to utilize the redundant forces (only one in this case) to improve the friction and the tipping conditions. To achieve this is we introduce additional object coordinates along a *virtual linkage*, thus removing the redundancy, and then constrain these coordinates by specifying the desired motions. This is analogous to the virtual linkage approach introduced by Williams and Khatib[20], [14] which determined internal forces using the virtual linkage. Note that such definition of *virtual linkage* is very different from the *virtual linkage* defined in [21] which is equivalent to our *p-robots*. We feel that the method presented here to handle redundant actuation can be useful to analyze multiple contact problems discussed in [21].

Introduction and constraining of additional virtual linkage coordinates provides an initial solution in the force space, but it may not be the most effective one to improve the friction and the tipping conditions. To find an optimal solution we determine the null hyperplane in the contact force space which actuates only the virtual linkage motions, but not the other object motions. This null hyperplane has the same dimension as the number of redundant virtual linkage coordinates. We can add actuation forces along this null hyperplane to the initial solution to achieve optimal friction and tipping conditions without affecting the desired object motions.

For 3 robots, we introduce a virtual linkage along the connection between robots A and B as shown in Figure 6. The new robot generalized coordinates are: $\mathbf{q_o}_N = [x_o, y_o, \theta_{o1}, \theta_{l11}, \theta_{l12}, \theta_{l21}, \theta_{l22}, x_{l1}]^T$. The kinematic constraints are:

$$J_{8\times8}\dot{\mathbf{q}}_{\mathbf{r}} = G_{8\times8}\dot{\mathbf{q}}_{\mathbf{o}} \tag{16}$$

where Jacobian matrix: J, and the grasp matrix: G, are as defined earlier. Note that the dimension of the grasp matrix is 8×8 and above relation gives 7 kinematic constraints, as well as constraints x_{l1} to zero. We can generate dynamic equations of motions for the system as derived in the previous section to arrive at equations 8 and 9.

We carry out the static analysis for the 3-robot system, similar to the 2-robot case, by solving the equation $(G^T)^{-1}\mathbf{n_o} = \mathbf{f_c}$. This leads to the exact same conditions for force, friction and tipping with robots B and C acting similar to the 2-robot case, and robot A just sits with no traction because the virtual linkage does not transmit force in the initial solution. This solution is not optimal because we are not taking advantage of the additional robot to distribute traction. To achieve this we determine the null hyperplane that actuates only the additional virtual linkage motion *i.e.* along x_{l1}. In this case the null force hyperplane has only 1-DOF since there is only one virtual link motion. The null force direction, ν_{force}, can be determined by setting all but the virtual linkage motions to zero:

$$\nu_{force} = (G^T)^{-1}_{8\times8}.[0,0,0,0,0,0,0,1]^T \tag{17}$$

So, redundant actuation forces can be applied along the direction ν_{force} without affecting object generalized coordinates other than the virtual linkage coordinates. Thus new set of the contact forces is given as:

$$\mathbf{f_c}_N = \mathbf{f_c} + K.\nu_{force} \tag{18}$$

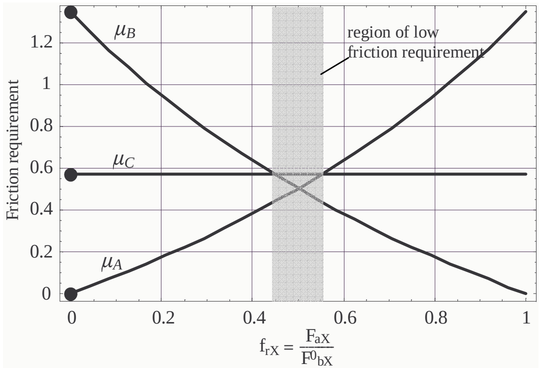


Fig. 7. Variation of the friction requirement on three robots as the contribution to the traction by robot A f_{rX} varies. The friction requirement curves for robots A and B intersect at $f_{rX} = 0.5$ giving an optimal solution, but the overall friction requirement is determined by μ_C in the highlighted region.

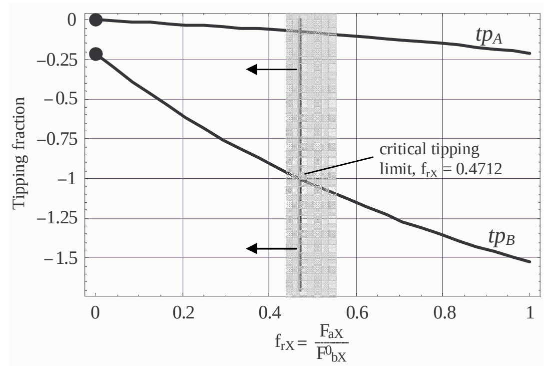
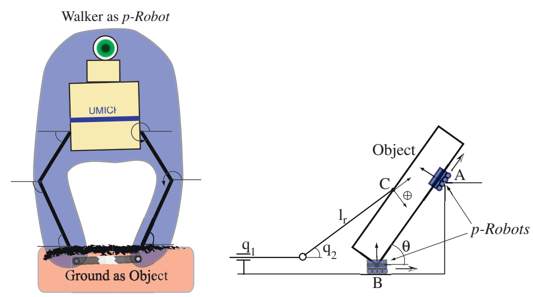


Fig. 8. Variation of the tipping condition on robots A and B as the contribution to the traction by the robot A f_{rX} varies. The tipping condition is satisfied under the redundant actuation as long as the contribution by robot A, defined by f_{rX}, is within the bound: $f_{rX} < 0.471$



(a) A simple walker (b) Block sliding example

Fig. 9. Presentation of preliminary ideas for applying our framework to other interesting multi-body robotic systems involving environmental interactions.

where K is an arbitrary parameter with units of force that we can vary to find the optimal distribution of traction. We scale K by the ratio between robot A traction force and the initial solution for the middle robot's traction force thus providing the relative distribution of traction force between A and B.

$$K = f_{rX}.F^{\circ}_{bX} \tag{19}$$

where f_{rX} is the fraction between 0 and 1 that represents the contribution to the traction force by robot A, and F°_{bX} is the nominal traction force required to be exerted by robot B for quasi-static lifting given by Equation 12.

$$f_{rX} = \frac{F_{aX}}{F^{\circ}_{bX}} \tag{20}$$

Thus f_{rX} provides a knob that we can turn to achieve optimal friction and tipping conditions.

1) Friction Requirements Under Redundant Actuation: We determine how the friction requirements on the robots vary as the contribution to the traction force by robot A, represented by f_{rX}, varies. Figure 7 gives the variation of the friction requirements on the three robots as f_{rX} varies. As the contribution of robot A increases, μ_A goes up and μ_B goes down. There is no change in μ_C since the contribution to traction by robot A does not affect robot C. The plots for μ_A and μ_B intersect giving the optimal friction requirement $\mu_A = \mu_B = 0.502$ at $f_{rX} = 0.5$ This means that an equal contribution to traction by robots A and B leads to minimum friction requirements on these two robots. In the highlighted region, as shown in Figure 7, around this optimal value, μ_A and μ_B are both lower than μ_C representing the useful range of force distribution between the two rear robots.

2) Tipping Conditions Under Redundant Actuation: As discussed previously, in the 2-robot static lifting case the tipping condition is satisfied by a large margin. We want to check that under this system of redundant actuation with three robots the tipping condition on robots A and B is still satisfied.

Figure 8 gives the variation of the tipping condition for robots A and B as f_{rX} varies. For $f_{rX} = 0$ (no contribution by robot A), the tipping fractions t_{pA} and t_{pB} are both well between ± 1, the critical limits for tipping. However, as the contribution from robot A increases, ($f_{rX} > 0$), the tipping fraction on robot B, tp_B, approaches -1 due to the moment generated by the two links, and at $f_{rX} = 0.471$, tp_B crosses the critical tipping limit. Thus the tipping condition is satisfied under the redundant actuation as long as the contribution by robot A, defined by f_{rX}, is within the bound $f_{rX} < 0.471$.

IV. Applications to Other Robotic Systems

In general, this framework is useful in analyzing any multi-body robotic system involving environmental interactions. The introduction of *p-robots* at points of contact with the environment 'opens up' closed kinematic chains which are cumbersome to analyze and decoupling achieved through constraining the *p-robots* facilitates analysis of kinematic as well as force constraints. In the future, we plan to apply this approach to analyze a variety of robotic systems including more complicated mobility systems and cooperative manipulation systems.

1) Walking Problem: To analyze walking robots such as the simple walker shown Figure 9(a) using our approach, the whole walker can be considered as a *p-robot* and the ground as an 'object' which is being manipulated. Then the walker's dynamics will need to be mirrored in the 'object' *i.e.* ground in the *p-robot*-fixed frame, and internal forces can be determined by introducing virtual linkages.

2) Block Standing Problem: Our approach can be extended to analyze cooperative manipulation systems, in which contact with the environment is critical. For example consider the system analyzed in [17], as shown in Figure 9(b). Instead of analyzing the contacts at points A and B as environmental constraints, as done in [17], we can introduce 2-DOF *p-robots* at these two points. We can then constrain the 'tangential' motions of the *p-robots* via kinematic constraints and the 'normal' motions to be zero. Such an approach has the benefit of being able to explicitly solve for the contact forces, thus allowing for evaluation not only of the allowable motions, but also of the required friction conditions for a desired motion.

V. Conclusions

This paper presents results obtained toward the development of a unified framework to represent physically cooperating mobile robots. We present the idea that a team of robots, physically cooperating to improve *mobility* can be considered as a *manipulation* system and we can consider that the cooperative maneuvers are achieved through actuation of passive internal degrees of freedom of the system via ground interactions. We demonstrate how the existing representations for manipulation systems can be extended and adapted to develop a unified framework for physically cooperating mobile robots.

By treating the linked bodies of the mobile robots as a multiple degree-of-freedom object comprising an articulated open kinematic chain, and by introducing the concept of virtual robots at the ground interaction points and virtual degrees of freedom of the contact points, this newly developed framework allows for a decoupled approach to analyzing a team of mobile robots. Dynamics of the open chain are computed independently of the constraints, thus allowing the same set of equations to be used as the constraint conditions change, and simplifying the addition of multiple robots to the chain. The constraints are represented in the space of the contacts, which is much simpler than doing so in the multi-body space of the manipulated chain. In addition, since contacts with the ground are treated as virtual robots, rather than kinematic constraints on the object, the representation allows for the direct solution of required ground forces and moments providing a direct evaluation of required friction and tipping conditions. We have introduced the idea of a 'tipping cone', similar to a standard friction cone, to test whether forces on the robots cause undesired tipping. We have carried out static as well as dynamic analysis for the 2-robot cooperation case, validating the new framework with comparison to a previous direct dynamic analysis. Also, we have demonstrated that introduction of redundant actuation, by an additional third robot, can help in improving the friction requirements. In this case, the framework provides easy means to add the third robot and a means of enumerating the redundant actuation degrees of freedom through the introduction of a virtual linkage.

The methodology presented here is by no means limited to 2-D, planar analysis or to simple cooperative behaviors. We have presented 2-D analyses for the sake of clarity and understanding. In our future work we will present 3-D analysis of more complicated behaviors.

In the future, we also plan to apply this approach to a variety of robotic systems including more complicated mobility systems and cooperative manipulation systems, and also to a wide variety of contact conditions and geometries. In this paper, we have presented our preliminary ideas in this direction. In a more general sense, this methodology applies to systems where actuated contact with the environment is used to drive unactuated internal degrees of freedom. This is complementary to typical mobility systems which we drive external degrees of freedom through actuated internal degrees of freedom through contact constraints. Our analysis suggests that an exchange of ideas and unification may be possible between these fields.

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